CONVERGENCE OF COMPUTATIONAL METHODS AND STABILITY
OF SELF-BALANCED STRESSES UNDER SHRINKAGE
OF SPHERICAL INCLUSIONS OF A DAMAGEABLE MATERIAL

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UDC 539.3:517.946

Some iterative methods for calculating self-balanced stresses under shrinkage of a ball inclusion enclosed in a spherical matrix of a physically nonlinear damageable material. The stability of this system was studied using methods of catastrophe theory. It has been established that the beginning of divergence of the proposed iterative processes coincides with the moment of transition of the system to an unstable position of equilibrium.

Key words: *stability, self-balanced stresses, damageable material.*

Introduction. Some composite manufacturing operations cause shrinkage of inclusions (filling agent), which is sometimes so large that the tensile strains of the inclusions reach values for which their material enters an unstable supercritical stage of deformation (softening stage) due to the occurrence of multiple defects. For a macroscopic description of the material behavior at this stage, the Drukker stability postulate does not apply [1]. We note that the strain softening property is inherent in structurally nonuniform materials, such as composites, concrete, geomaterials, etc., [2–4].

The occurrence of regions of physically unstable material can make the equilibrium position of the entire system unstable, and under an infinitesimal perturbation, the system suddenly enters the nearest stable position of equilibrium. As a result of this jump, the inclusions can be damaged.

Apparently, a convergent iterative process of calculating self-balanced stress fields determines the stable position of equilibrium of the system. If the process diverges, relationship of this phenomenon with the physical state of the body, as a rule, is difficult to establish.

In the present paper, we consider a number of iterative procedures for calculating self-balanced stresses arising from shrinkage of a spherical inclusion of a physically nonlinear material enclosed in a spherical matrix.

1. Computational Model and Properties of Material. We consider a composite consisting of an elastic matrix and a filling agent in the form of spherical inclusions of a nonlinear material. After manufacture of the composite, the inclusions are subjected to shrinkage determined by the negative volume strain e^* in the absence of relationships of the inclusions with the spherical region. Because the enclosing matrix prevents the inclusions from varying in volume, a self-balanced field of residual stresses arises in the composite and the inclusions are stretched. If the inclusions are far apart, the stress and strain fields near them hardly influence each other. In this case, the computational model consists of an inner sphere (inclusion) of radius a (region V_a) and a an enclosing thick-walled sphere (matrix) of outer radius b (region V_b). We assume that under loading, the material of the sphere always remains in an elastic state with Young's modulus E, Poisson's constant ν, and bulk modulus K. The material of the inclusion have the same characteristics at the elasticity stage.

In the computational model at hand, shrinkage of the inner sphere gives rise to uniform extension of the inner sphere. Therefore, to describe the properties of the material, it suffices to obtain a complete p–e stress–strain diagram (p is the volumetric stress and e is the volumetric strain) (Fig. 1). On the diagram, p^t, p^b, e^t, and e^b are the proportional and strength limits for uniform extension and corresponding volumetric strains and
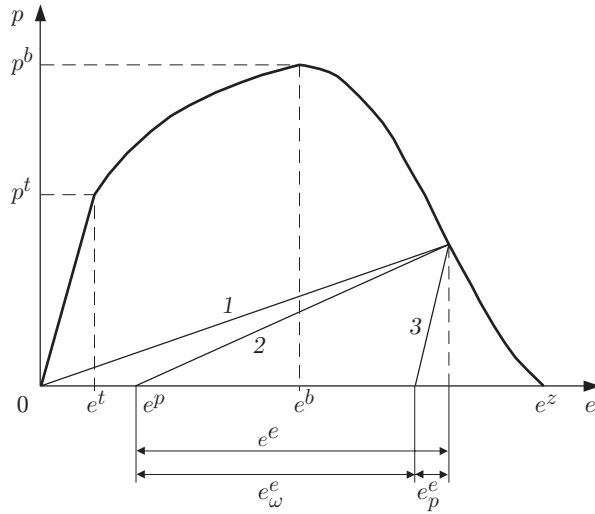
Fig. 1

e^z is the failure strain. At the hardening stage (ascending branch of the p–e diagram after the proportional limit) and softening stages (descending branch), the properties of the material determine the secant (instantaneous) bulk modulus $K^p = dp/de$ ($K^p < K$).

In uniform extension, macroscopic plastic strains do not arise. Therefore, the nonlinearity of the $p(e)$ diagram is due to damage of the material by microcracks, micropores, etc. In this case, two versions of unloading are possible: 1) without residual strain with the secant modulus $K^s = p/e$ (line 1 in Fig. 1), where the damage is absolutely brittle; 2) with formation of insignificant residual strains e^p and unloading modulus $K^u = p/(e - e^p)$, $K^u < K$ (line 2 in Fig. 1) if near the fracture, for example, at the microcrack tips, local microplastic regions form.

Using a similar line of reasoning as in [5, 6], we obtain the relation

$$p = K(1 - \omega)(e - e^p), \tag{1.1}$$

where the parameter ω characterizes the damage of the material by microdefects:

$$\omega = 1 - K^u/K. \tag{1.2}$$

Using expression (1.1), we obtain

$$dp = K(1 - \omega)(de - de^p) - K(e - e^p)\, d\omega.$$

At the same time, the relation for increments $dp = K^p\, de$ is valid. Hence,

$$(e - e^p)\frac{d\omega}{de} + \omega\left(1 - \frac{de^p}{de}\right) = 1 - \frac{K^p}{K} - \frac{de^p}{de}. \tag{1.3}$$

If residual deformations do not arise, formula (1.3) becomes

$$e\frac{d\omega}{de} + \omega = 1 - \frac{K^p}{K}. \tag{1.4}$$

Equation (1.3) defines the damage residual strain kinetics and Eq. (1.4) defines the damage kinetics in the absence of residual strains. Direct check shows that expression (1.2) is a solution of Eq. (1.3), and for $K^u = K^s$, it is a solution of Eq. (1.4).

After multiplication by de, equality (1.3) can be written as $de_\omega^e = d[\omega(e - e^p)] = de_\omega^p - de^p$, where

$$de_\omega^p = (1 - K^p/K)\, de \tag{1.5}$$

is the increment of the pseudoplastic strain in the case $K^u = K$ (line 3 in Fig. 1). A similar representation holds for expression (1.4): $de_\omega = de_\omega^p$, where $e_\omega = \omega e$.

The decomposition of the total strain e into the components e^p, $e_p^e = p/K$, $e^e = p/K^u = e - e^p$, and $e_\omega^e = \omega e^e$ used in the above reasoning is shown in Fig 1.

Writing Eq. (1.1) with the use of the indicated strain components, we obtain

$$p = K[e - e^p - \omega(e - e^p)] = K[e - (e^p + e^e_\omega)] = K(e - e^p_\omega), \tag{1.6}$$

$$p = K[e - e^p - \omega(e - e^p)] = K[e - (e^p + e^e_\omega)] = K(e - e^p_\omega), \tag{1.6}$$

where e^p_ω is total pseudoplastic strain determined by unloading with modulus K. Its variation depends on variations in e^p and ω and is described by Eq. (1.3) [or (1.4)] and, formally, by equality (1.5).

2. Initial-Boundary-Value Problem. To obtain the governing relation in the region V_a taking into account its shrinkage, we assume that the inner sphere is free of constraints and specify a shrinkage strain e^*. As a result, the points of the boundary perform radial displacements directed to the center of the sphere and equal to e^*a. We again unite the regions V_a and V_b. After establishment of equilibrium, the points of the inner boundary of the region V_b perform displacements $e''a$ directed from the center. Obviously, $e^*a + e''a = e'a$. Therefore, the sphere V_a is subjected to a volumetric tensile strain e'' related to the stress p'' by relation (1.1). Thus, in the region V_a, the following relation is valid:

$$p'' = K(1 - \omega)(e'' - e^p) = K(1 - \omega)(e' - e^* - e^p). \tag{2.1}$$

In the region V_b, in which shrinkage is absent, the following equalities holds:

$$\sigma''_r = E[(1 - \nu)\varepsilon'_r + 2\nu\varepsilon'_\theta]/(mn), \qquad \sigma''_\theta = E(\nu\varepsilon'_r + \varepsilon'_\theta)/(mn),$$

$$\sigma''_\varphi = \sigma''_\theta, \qquad \varepsilon'_\varphi = \varepsilon'_\theta. \tag{2.2}$$

Here $m = 1 + \nu$, $n = 1 - 2\nu$, σ_r, σ_θ, σ_φ, ε_r, ε_θ, and ε_φ are the stress and strain in spherical coordinates, whose origin coincides with the center of the inner sphere; two primes denote self-balanced stresses and one prime denotes strains that satisfy the compatibility conditions.

Relations (2.1) and (2.2) should be supplemented by the equilibrium equation

$$\frac{d\sigma''_r}{dr} + 2\frac{\sigma''_r - \sigma''_\theta}{r} = 0,$$

Cauchy relations

$$\varepsilon'_r = \frac{du}{dr}, \qquad \varepsilon'_\theta = \frac{u}{r}$$

(u is the radial displacement), and the boundary conditions $\sigma''_r|_{r=b} = 0$ (the outer boundary is free of stresses) and $u|_{r=0} = 0$ (the central point does not move). As a result, for specified e^*, e^p, and ω, we have a closed system of equations for determining the stress–strain states

3. Iterative Process Using Pseudoplastic Strain. The difficulty in solving the problem formulated above lies in the fact that the strain e^p and damage ω are not known beforehand. Their values depend on the strains, which, in turn, depend on e^p and ω. Therefore, we deal with a coupled boundary-value problem. To solve this problem, one should employ special iterative methods.

Let us consider a method based on the use of pseudoplastic strain. Deriving the governing relation for the region V_a using formula (1.6), we obtain the relation

$$p'' = K(e' - e^* - e^p_\omega).$$

Let us split the initial problem into a primal problem with the governing law

$$p'' = K(e' - e^* - e^p_\omega)$$

and a correcting problem with the law

$$p'' = K(e' - e^*).$$

To these relations, we add formulas (2.2) for the region V_b, equilibrium equations, Cauchy relations, and the boundary conditions given above. In addition, the correcting problem is supplemented by the kinetic equation (1.5).

Solution of the primal problem defines the stress–strain state in an absolutely elastic compound sphere with shrinkage of the inclusion. Solution of the correcting problem yields the stress–strain state in an elastic body with the inner sphere having a residual plastic strain e^p_ω. Apparently, for specified values of e^* and e^p_ω, the solution of the original problem is the sum of the solutions of the primal and correcting problems.
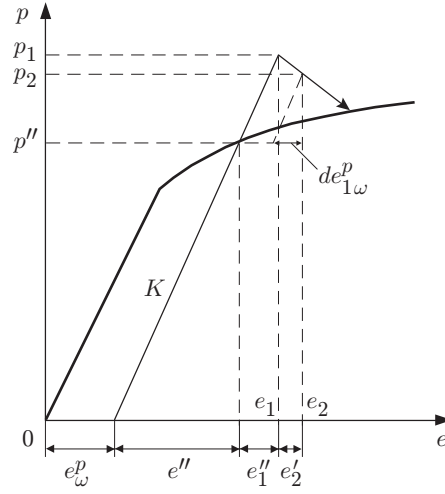
Fig. 2

The general form of the solution of the primal problem is given by the formulas

$$p'' = K(e' - e^*), \qquad e' = Ae^*, \qquad u = Ae^*r, \qquad r \in V_a,$$
$$\sigma_r'' = Me^*(1 - b^3 r^{-3}), \qquad \sigma_\theta'' = \sigma_\varphi'' = Me^*(1 + 0.5b^3 r^{-3}),$$
$$\varepsilon_r' = Ne^*(n - mb^3 r^{-3}), \qquad \varepsilon_\theta' = \varepsilon_\varphi' = Ne^*(n + 0.5mb^3 r^{-3}), \tag{3.1}$$
$$u = Ne^*(nr + 0.5mb^3 r^{-2}), \qquad r \in V_b.$$

Using the continuity property of radial stresses and displacements, we find

$$M = \frac{2a^3 E}{b^3(m + 2n)}, \qquad A = \frac{b^3 m + 2a^3 n}{b^3(m + 2n)}, \qquad N = \frac{M}{E}.$$

The solution of the correcting problem for a specified value of e_ω^p is also defined by formulas (3.1) after replacement of the strain e^* by e_ω^p.

We assume that for a certain value of $e^* < 0$, the composite sphere is in equilibrium and in the region V_a, the pseudoplastic strain, secant bulk modulus, unloading modulus, and damage are equal to e_ω^p, K^p, K^u, ω, respectively. In the initial position, the stress–strain state in the region V_a is defined by the scalars $e = e_\omega^p + e''$ and $p'' = Ke''$ (Fig. 2), and in the region V_b, it is defined by the strain vector function $\boldsymbol{\varepsilon}' = (\varepsilon_r', \varepsilon_\theta', \varepsilon_\varphi')$ and stresses $\boldsymbol{\sigma}'' = (\sigma_r'', \sigma_\theta'', \sigma_\varphi'')$. Thus, the stress–strain state of the entire compound sphere can be defined by the following pairs of scalars and vectors: $\{e, \boldsymbol{\varepsilon}'\}$, $\{p'', \boldsymbol{\sigma}''\}$.

We subject the inclusion to an additional small shrinkage Δe^* ($\Delta e^* < 0$). Using formulas (3.1) and substituting e^* by Δe^*, we obtain a solution of the primal problem: p_1'', $e_1'' = e_1' - \Delta e^*$, ε_1', $\boldsymbol{\sigma}_1''$. In this case, the expressions $\{p_1, \sigma_1\} = \{p'', \boldsymbol{\sigma}''\} + \{p_1'', \boldsymbol{\sigma}_1''\}$ and $\{e_1, \varepsilon_1\} = \{e, \boldsymbol{\varepsilon}'\} + \{e_1'', \varepsilon_1'\}$ represent the first approximation to the solution of the original problem for the shrinkage strain $e^* + \Delta e^*$. Then, for the value of e_1, we determine the characteristics of the material $K_1^p = K^p(e_1)$, $K_1^u = K^u(e_1)$, and $\omega_1 = 1 - K_1^u/K$ in the region V_a.

Next, substituting the quantities $de = e_1''$ and K^p into relation (1.5), we calculate $de_{1\omega}^p$. Setting $e^* = de_{1\omega}^p$ in formulas (3.1), we obtain a solution of the correcting problem $\{e_2', \varepsilon_2'\}$, $\{p_2'', \boldsymbol{\sigma}_2''\}$ and find the second approximation $\{p_2, \sigma_2\} = \{p_1, \sigma_1\} + \{p_2'', \boldsymbol{\sigma}_2''\}$, $\{e_2, \varepsilon_2\} = \{e_1, \varepsilon_1\} + \{e_2', \varepsilon_2'\}$. For the value of e_2, we have $K_2^p = K^p(e_2)$, $K_2^u = K^u(e_2)$ and $\omega_2 = 1 - K_2^u/K$. Again, using formulas (1.5), where $de = e_2'$ and $K^p = K_1^p$, we calculate the increment $de_{2\omega}^p$; using formulas (3.1), where $e^* = de_{2\omega}^p$, we obtain the following correcting solution and find the third approximation, etc. This iterative process is shown schematically in Fig. 2.

The convergence of the method determines the convergence of the iterative series for strains in the region V_a. We have

$$e = e'' + (A - 1)\Delta e^* + \sum_{i=0}^{\infty} C_i, \tag{3.2}$$

where $C_0 = A(1 - K_0^p/K)$, $C_i = B_i C_{i-1}$, $K_0^p = K^p$, and $B_i = A(1 - K_i^p/K)$. The numerical series with positive terms (3.2) converges for $B_i < 1$. If at the jth step, $B_j = B_* = 1$ and, hence, the secant modulus reaches the value
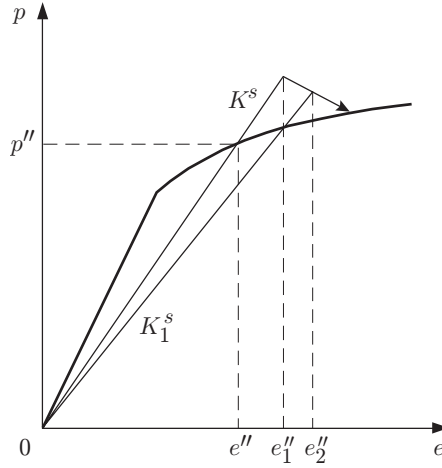
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Fig. 3

$$K_*^p = -K\,\frac{2n(b^3 - a^3)}{b^3 m + 2a^3 n}, \tag{3.3}$$

two cases are possible. In the first case, $K_*^p = \min\limits_e K^p$. Then, at all subsequent steps, $B < 1$. In the second case,

$$K_*^p > \min\limits_e K^p, \tag{3.4}$$

and at the next steps, $B > B_*$, i.e., the terms of the series begin to increase. Thus, equality (3.3) is a necessary condition, and in the aggregate with inequality (3.4), it is a necessary and sufficient condition that defines the beginning of divergence of the iterative process.

After reaching the minimum, the modulus K^p increases, tending to the limiting value $K^p(e^z) = 0$. Therefore, at a certain step, the equality $B = B_*$ is reached again, and from this moment, the series (3.2) becomes convergent.

We note that for the calculated position of equilibrium in the region V_a, the values of the material parameters K^p, K^u, and ω are equal to the values obtained at the last iterative step.

4. Iterative Method for a Material with Absolutely Brittle Damage. Using law (2.1) for $e^p = 0$, namely

$$p'' = K^s(e' - e^*) \tag{4.1}$$

[$K^s = K(1 - \omega)$ is the secant modulus], we construct the corresponding iterative procedure.

In the initial positions of equilibrium described above, we replace K^u by K^s and e by $e'' = e' - e^*$ (Fig. 3). We perturb these positions by specifying an additional shrinkage Δe^*. Considering the system elastic, we find the stress–strain state for the total shrinkage $e_1^* = e^* + \Delta e^*$. It is given by formulas (3.1) after replacement of the governing relation in the region V_a by law (4.1) and the strains e^* by e_1^*. The unknown constants are obtained using the continuity of radial displacements:

$$M = M_1(K^s) = 2a^3 K^s(e'')EH^{-1}, \quad A = A_1(K^s) = K^s(e'')hH^{-1}, \quad N = N_1 = M_1 E^{-1}.$$

Here $h = 2a^3 n + mb^3$ and $H = K^s(e'')h + 2E(b^3 - a^3)$. For the value of $e_1'' = [A_1(K^s) - 1]e_1^*$ in the region V_a, we determine the new characteristics of the material $K_1^p = K^p(e_1'')$, $K_1^s = K^s(e_1'')$, and $\omega_1 = 1 - K_1^s/K$ and solve the elastic problem with these parameters. We have $M = M_1(K_1^s)$ and $A = A_1(K_1^s)$. From the strains $e_2'' = [A_1(K_1^s) - 1]e_1^*$, we obtain the characteristics $K_2^p = K^p(e_2'')$, $K_2^s = K^s(e_2'')$, and $\omega_2 = 1 - K_2^s/K$ and again solve the elastic problem, etc. This iterative procedure is shown schematically in Fig. 3; it represents the simple iterative scheme

$$e_{n+1}'' = \psi(e_n''),$$

where $\psi(e_n'') = [A_1(K^s(e_n'')) - 1]e_1^*$. It is known that the method converges if the function ψ satisfies the Lipschitz condition

$$|\psi(e_{n+1}'') - \psi(e_n'')| < k|e_{n+1}'' - e_n''|$$

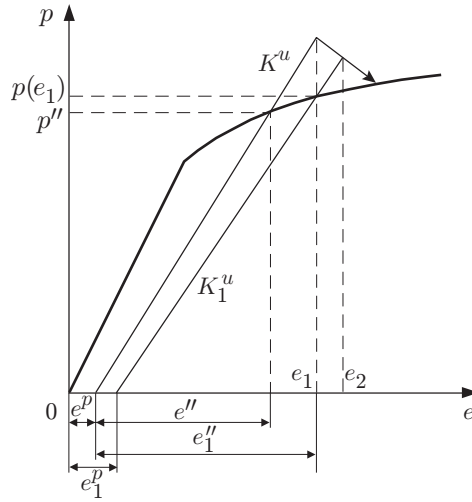with the constant $k < 1$ [7]. In this case, the principle of compressing maps holds.

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Fig. 4

The compressibility condition is satisfied if $\sup |d\psi/de''| < 1$. Differentiating the function ψ taking into account that $dK^s/de'' = (K^p - K^s)/e''$, we obtain

$$\frac{d\psi}{de''} = 2E(b^3 - a^3)e_1^* h(K^p - K^s)(e'')^{-1}H^{-2}. \tag{4.2}$$

Because of the continuity of displacements at $r = a$, the following equality should hold in the position of equilibrium:

$$e'a = M_1 e_1^*(na + 0.5mb^3 a^{-2})E^{-1}.$$

Taking into account that $e' = e'' + e_1^*$ and performing appropriate transformations, we obtain

$$He'' = -2E(b^3 - a^3)e_1^*.$$

Using this equality and adding and subtracting the term $2E(b^3 - a^3)$ in the numerator in expression (4.2), we obtain

$$\frac{d\psi}{de''} = 1 - \frac{K^p h + 2E(b^3 - a^3)}{H}.$$

From analysis of this expression, it follows that the Lipschitz condition is not satisfied when the instantaneous modulus K^p reaches the value K_*^p [see (3.3)].

Thus, in this case, too, satisfaction of condition (3.3) is also necessary for the beginning of divergence of the iterative process. Following the same line of reasoning as above, we find that the aggregation of equality (3.3) and inequality (3.4) is a necessary and sufficient condition for the beginning of divergence of the process. It is also obvious that, eventually, the process converges because the modulus K^p necessarily reaches values larger than K_*^p.

5. Iterative Method for Arbitrary Fracture. We construct an iterative procedure for solving the problem with the general law (2.1), in which we designate $K(1 - \omega) = K^u$. In the initial position of equilibrium with shrinkage e^* in the region V_a, the secant bulk modulus, unloading modulus, residual strain, total strain, and stress are equal to K^p, K^u, e^p, $e = e^p + e''$, and $p'' = K^u e''$, respectively (Fig. 4). We perturb this position of equilibrium by specifying an additional shrinkage Δe^*.

Fixing the value of e^p, we determine the stress–strain state with shrinkage $e_1^* = e^* + \Delta e^*$. It is evaluated from formulas (3.1), in which K is replaced by K^u and e^* by the sum $e_1^* + e^p$. Determining the values of the constants from the continuity condition for displacements and radial stresses, we have $A = A_1(K^u)$ and $M = M_1(K^u)$. Then, $e_1'' = (A_1 - 1)(e_1^* + e^p)$, and the total strain in the region V_a equals $e_1 = e^p + e_1''$ (Fig. 4). For the value of e_1, we find $K_1^u = K^u(e_1)$, $K_1^p = K^p(e_1)$, $\omega_1 = 1 - K_1^u/K$, and the total residual strain $e_1^p(e_1) = e_1 - p(e_1)/K_1^u$. Fixing the value of e_1^p, we again use formulas (3.1), where $A = A_1(K_1^u)$ and $M = M_1(K_1^u)$. We have $e_2'' = (A_1 - 1)(e_1^* + e_1^p)$, $e_2 = e_1^p + e_2''$, $K_2^u = K^u(e_2)$, $K_2^p = K^p(e_2)$, $\omega_2 = 1 - K_2^u/K$, and $e_2^p(e_2) = e_2 - p(e_2)/K_2^u$. Further calculations are repeated. This iterative procedure is shown schematically in Fig. 4 and represents the simple iterative scheme
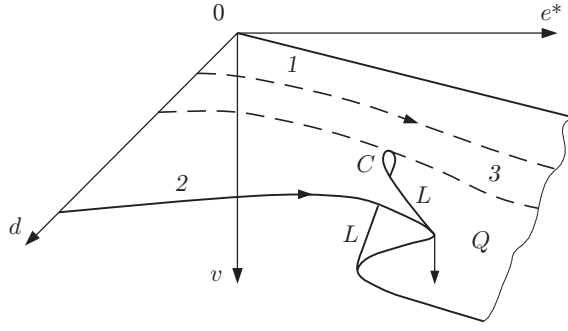
$$e_{n+1} = \varphi(e_n),$$

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Fig. 5

where $\varphi(e_n) = A_1(K^u(e_n))(e_1^* + e_n^p(e_n)) - e_1^*$. Let us check satisfaction of the Lipschitz condition. We obtain

$$\frac{d\varphi}{de} = \frac{d}{de}\Big(\frac{K^u(e)h(e_1^* + e^p(e))}{H(K^u)}\Big) = \Big[\frac{dK^u}{de}h(e_1^* + e^p)2E(b^3 - a^3) + K^u hH\frac{de^p}{de}\Big]H^{-2}. \tag{5.1}$$

The expression for de^p/de is obtained from formula (1.3) after substitution of the value of ω from formula (1.2):

$$\frac{de^p}{de} = 1 - \frac{K^p}{K^u} + \frac{e - e^p}{K^u}\frac{dK^u}{de}. \tag{5.2}$$

In the position of equilibrium, the following equality should also be satisfied:

$$e'a = M_1(e_1^* + e^p)(na + 0.5mb^3a^{-2})E^{-1}. \tag{5.3}$$

Here $M_1 = 2a^3K^u(e)E/H(K^u)$ and $e' = e_1^* + e$ ($e_1^* < 0$, $e > 0$). Then, expression (5.3) leads to

$$He = K^u e^p h - 2E(b^3 - a^3)e_1^*. \tag{5.4}$$

Substituting expressions (5.2) and (5.4) into formula (5.1) and performing appropriate transformations, we obtain

$$\frac{d\varphi}{de} = \frac{K^u - K^p}{H}h = \frac{K^u h + 2E(b^3 - a^3) - K^p h - 2E(b^3 - a^3)}{H} = 1 - \frac{K^p h + 2E(b^3 - a^3)}{K^u h + 2E(b^3 - a^3)}.$$

From analysis of this expression, it follows that the Lipschitz condition is not satisfied for $K^p = K_*^p$ [see (3.3)]. This condition is necessary for the beginning of divergence of the process. By the same reasoning as above, we elucidate that the aggregation of equality (3.3) and inequality (3.4) is a necessary and sufficient condition for the beginning of divergence of the process. We note that, eventually, the process converges because the modulus K^p necessarily reaches values larger than K_*^p.

6. Stability of Self-Balanced Stresses. To determine the physical sense of the obtained divergence conditions of the iterative processes considered above, we study the stability of the self-balanced stress field resulting from shrinkage. Considering the compound sphere as a discrete mechanical system, we can assume that the shrinkage strain e^* is a specified control parameter and the displacement of the boundary points of the inner sphere v is a state variable of the system. The total energy of the sphere is written as a two-parameter function of the control and state parameters:

$$\Pi(v, \varepsilon^*) = 4\pi a^2\int\limits_0^v p(v/a)\,dv + 4\pi a\frac{Kn}{h}(b^3 - a^3)w^2,$$

where the first term is the strain energy of the inner sphere for displacement of the points of its boundary at a distance v and the second term is the potential energy of elastic strains of the second sphere for displacement of the inner boundary at a distance $w = e^*a + v$ provided that the outer boundary is free of stresses.

It is known that the critical points of the function Π determined from the solution of the equation

$$\frac{d\Pi}{dv} = 4\pi a^2 p(v/a) + 8\pi a\frac{Kn}{h}(b^3 - a^3)(e^*a + v) = 0, \tag{6.1}$$

specify all positions of equilibrium of the system [8]. In three-dimensional space with the coordinate axes e^*, v, and d ($d = b - a$), the aggregation of this point forms the surface Q — the so-called Whitney cusp-catastrophe [8] (Fig. 5). The doubly degenerate points at which stable positions of equilibrium becomes unstable and vice versa

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are determined by simultaneous solution of Eq. (6.1) and the equation obtained by setting the determinant of the Hess matrix of the function Π equal to zero [8]:

$$\frac{d^2\Pi}{dv^2} = K^p + 2K(b^3 - a^3)\frac{n}{h} = 0, \tag{6.2}$$

also form assembly lines L. The triply degenerate stagnation point is determined by solving Eqs. (6.1) and (6.2) and the equation

$$\frac{d^3\Pi}{dv^3} = \frac{dK^p}{dv} = \frac{dK^p}{de}\frac{1}{a} = 0. \tag{6.3}$$

This point is the assembly origin (point C in Fig. 5).

If the value of d is small, the trajectory of the equilibrium states on the surface Q does not intersect the assembly line (line 1 in Fig. 5) as the control parameter e^* increases and all the equilibrium positions of the stress fields are stable. If the trajectory of equilibrium states on the surface Q intersects the assembly line (line 2 in Fig. 5), the system loses stability and suddenly enters a new position of equilibrium (trajectory from the upper sheet of the surface "springs" onto the lower sheet). Therefore, for the control parameter value corresponding to the point of intersection of the assembly line, the self-balanced stress state is unstable and its any perturbation by an additional small shrinkage results in a loss of stability. Solving Eq. (6.2), we find that at the given moment, condition (3.3) is satisfied, i.e., $K^p = K_*^p$. Since Eq. (6.3) is not satisfied in this case, inequality (3.4) holds. If the trajectory of equilibrium states on the surface Q passes through the point C (line 3 in Fig. 5), loss of stability does not occur. The point C is determined by simultaneous solution of Eqs. (6.2) and (6.3). Hence it follows that $K^p = K_*^p$ is minimal and inequality (3.4) is not satisfied.

Thus, the physical sense of the divergence conditions of the iterative processes lies in the fact that the beginning of divergence corresponds to a loss of stability of the equilibrium of the compound sphere and the self-balanced stress field perturbed by an additional small shrinkage (equilibrium positions) is unstable.

REFERENCES

1. Yu. N. Rabotnov, *Mechanics of Deformable Solids* [in Russian], Nauka, Moscow (1988).
2. L. P. Khoroshun and E. N. Shikula, "Deformation of composite materials under microfractures," *Prikl. Mekh.*, **32**, No. 6, 52–58 (1996).
3. G. Frantziskonis and C. S. Desai, "Constitutive model with strain softening," *Int. J. Solids Struct.*, **23**, No. 6, 733–750 (1987).
4. I. V. Baklashov, *Deformation and Fracture of Rock Masses* [in Russian], Nedra, Moscow (1988).
5. V. V. Struzhanov and S. V. Zhizherin, "Model of a damageable material and iterative methods for calculating stress states under torsion," *Vychisl. Tekhnol.*, **5**, No. 2, 92–104 (2000).
6. S. V. Zhizherin, V. V. Struzhanov, and V. I. Mironov, "Iterative computational methods of calculating stresses for a pure bending of a beam of a damageable material," *Vychisl. Tekhnol.*, **6**, No. 5, 24–33 (2001).
7. A. N. Kolmogorov and S. V. Fomin, *Elements of Function Theory and Functional Analysis*, [in Russian] Nauka, Moscow (1989).
8. R. Gilmor, *Catastrophe Theory for Scientists and Engineers*, Book 1, Wiley, London (1981).